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COMMENT

Comment on 'Hannay angle in an *LCR* circuit with time-dependent inductance, capacity and resistance'

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Abstract
This practical example of angle holonomy has a mis-stated derivation which is rectified here lest it discredit the results obtained, which need no amendment.

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The dynamical system considered is a linear electrical circuit with a time-dependent inductance $L(t)$, capacitance $C(t)$ and resistance $R(t)$ in series. The equation governing the charge Q which has passed is, as stated in [1],

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(L(t)\frac{\mathrm{d}Q}{\mathrm{d}t}\right) + R(t)\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{Q}{C(t)} = 0$$

or defining $P \equiv L\,\mathrm{d}Q/\mathrm{d}t$, the equivalent first-order equations are

$$\begin{pmatrix} \dot{Q} \\ \dot{P} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{R}{L} \end{pmatrix} \begin{pmatrix} Q \\ P \end{pmatrix}.$$

Because R is present, representing dissipation, the matrix here has non-zero trace, that is, it violates Liouville's theorem that the flow in phase space (Q, P) is divergenceless. Consequently, since Hamilton's equations imply Liouville's theorem, there is no Hamiltonian leading to this equation (contradicting equation (3) of [1]). If R were zero the Hamiltonian would be $H(Q, P) = P^2/2L + Q^2/2C$.

Dissipative dynamical systems can exhibit angle holonomy of a kind though, as was shown by Kepler and Kagan [2] who analysed the phenomenon for a limit cycle. For a linear system such as the present one there is no limit cycle, only an attracting limit point at the origin in phase space. Nevertheless a clear, albeit non-canonical, angle exists, namely that of the phase point around the origin. To represent it in a conventional context one can introduce an artificial isotropic dilation of phase space by defining new coordinates $p = \exp\left(\int \frac{R}{2L}\,\mathrm{d}t\right)P$ and $q = \exp\left(\int \frac{R}{2L}\,\mathrm{d}t\right)Q$. This dilation keeps unchanged the angle of a phase point about the origin. This is may be the intention in equation (4) of [1], but the signs of the two exponents needed are the same, not opposite, specifically so that the transformation is a dilating one, not canonical.

The new coordinates q and p obey the equations

$$\begin{pmatrix} \dot{q} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} \frac{R}{2L} & \frac{1}{L} \\ -\frac{1}{C} & -\frac{R}{2L} \end{pmatrix} \begin{pmatrix} q \\ p \end{pmatrix}.$$

This matrix is traceless, so the phase flow in the artificial phase space (q, p) is divergence free and is generated by the Hamiltonian $H(q, p) = q^2/2C + Rqp/2L + p^2/2L$. This is analogous to equation (4) of [1] and the usual formalism can be invoked leading to the stated results for angle holonomy. The rectified derivation just given has not required any amendment of the results, which stand as given in [1].

In making this correction I might remark incidentally that in the same issue of *J. Phys. A: Math. Gen.* as the original letter [1], I myself stand corrected on a point of faulty reasoning also concerning holonomy [3]. Once again there, the final results need no amendment.

References

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